

# Color Dynamics In Phase Space: The Balescu-Lenard-Vlasov Approach

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We propose to use the Balescu-Lenard-Vlasov (BLV) equation to describe relativistic heavy ion collisions. We use an inter-quark Richardson's potential consistent with the indications of Lattice QCD calculations. The color degrees of freedom are explicitly taken into account. We explicitly demonstrate that the Vlasov approach alone is insufficient in the hadronization region. In order to overcome this problem we prepare the initial condition for many events using molecular dynamics with frictional cooling and a Thomas-Fermi approximation to the Fermi motion. These events are averaged and propagated in time using the Vlasov approach. The fluctuations are used to evaluate the collision term and in turn the number of  $q\bar{q}$  created.

## 1. INTRODUCTION

One of the open problems in theoretical nuclear and particle physics is how to obtain the well known nuclear properties starting from the quark degrees of freedom [1]. This also includes the possibility of understanding the basic free nucleon-nucleon interaction from quark and gluon dynamics. Some kind of solution to this problem is becoming more and more needed with the new experiments done or planned using ultra-relativistic heavy ions (rhic) at CERN and at RHIC. The search for a quark-gluon plasma (QGP) in such collisions is in fact one of the new and most exciting directions in physics at the border between nuclear and particle physics [2]. Quantum ChromoDynamics (QCD) because of its difficulties (numerical and conceptual), has been applied so far to some limited cases such as quark matter at zero baryon ( $\rho_B$ ) density and high temperature (T) [1,2]. Furthermore in rhic dynamics plays surely an important role and accordingly the theory should be dynamical.

Recently [3], we have proposed a dynamical approach based on the Vlasov equation [4,5] to reproduce hadron masses and the properties of nuclear matter at finite  $\rho_B$ . Some works in the same spirit are discussed in [6,7]. Our approach needs as inputs the interaction potential among quarks, which was borrowed from phenomenology i.e. the Richardson's potential[8], and the quark masses which were fitted to reproduce known meson masses such as the  $\pi$ , the  $\phi$ , the  $\eta_c$  etc. When the particles are embedded in a dense medium such as in nuclear matter (NM) the potential becomes screened in a similar fashion as ions and electrons in condensed matter do, i.e. Debye screening (DS)[2,5].

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Here we refine that approach in two important aspects which are the treatment of the color degrees of freedom and the inclusion of the collision term. In the previous works [3] color degrees of freedom were implicitly taken into account through the use of a Debye radius that effectively screens the  $qq$  interaction potential. In the present paper we give to the quarks explicitly a color (using the Gell-Mann matrices) and follow their dynamics in phase space solving the Vlasov equation (VE). Thus screening will be dynamically obtained. In general the self screening obtained in Vlasov dynamics is inadequate, which is the reason why it was not adopted in the earlier attempts [3]. In fact we will show explicitly that the Vlasov approach alone gives a good description of the system at large densities only, i.e. in the QGP region. In order to overcome this problem we adopted the following strategy. We first prepare the initial conditions using molecular dynamics (MD) with frictional cooling for many events. The events are averaged and care is taken of antisymmetrization. These are the initial conditions for the Vlasov evolution. Since the VE fulfills the Liouville theorem, the initial phase space density remains constant in time. Thus the initial antisymmetrization and eventual clustering obtained in the cooling process are maintained. When simulating a nucleus-nucleus collision, inter quark correlations and particle production become very important. We have included this feature using the most general form of the collision term which is the Balescu-Lenard type. This is very crucial since we are dealing with particles which are interacting through long range forces.

## 2. The Balescu Lenard Vlasov equation

We outline our approach on purely classical grounds, however the same results can be obtained within the Wigner transform formalism [4,5] of the quantum BBGKY-hierarchy in the limit  $\hbar \rightarrow 0$ .

The exact (classical) one-body distribution function  $f_1(r, p, t)$  satisfies the equation (BBGKY hierarchy)[5]:

$$\partial_t f_1 + \frac{\vec{p}}{E} \cdot \nabla_r f_1 = \int d(2) \nabla_r V(\mathbf{r}, \mathbf{r}_2) \nabla_p f_2(\mathbf{r}, \mathbf{r}_2, \mathbf{p}, \mathbf{p}_2, t) \quad (1)$$

$E = \sqrt{p^2 + m_i^2}$  is the energy and  $m_i = 10 \text{ MeV}$  is the (u,d) quark mass. Here we assume the potential to be dependent on the relative coordinates only. A generalization to include a momentum dependent part is straightforward.  $f_2$  is the two-body distribution function, which in the classical limit reads:

$$f_2(r, r_2, p, p_2, t) = \Sigma_{\alpha \neq \beta}^Q \delta(\mathbf{r} - \mathbf{r}_\alpha) \delta(\mathbf{p} - \mathbf{p}_\alpha) \times \delta(\mathbf{r}_2 - \mathbf{r}_\beta) \delta(\mathbf{p}_2 - \mathbf{p}_\beta) \quad (2)$$

where  $Q = q + \bar{q}$  is the total number of quarks and anti quarks. Inserting this equation into Eq.(1) gives:

$$\partial_t f_1 + \frac{\vec{p}}{E} \cdot \nabla_r f_1 - \nabla_r U \cdot \nabla_p f_1 = 0 \quad (3)$$

Where  $U = \Sigma_j V(\mathbf{r}, \mathbf{r}_j)$  is the exact potential. Let us now define  $f_1$  and  $U$  as sums of an ensemble averaged quantity plus the deviation from this average:

$$f_1 = \bar{f}_1 + \delta f_1; U = \bar{U} + \delta U \quad (4)$$

Substituting these equations in Eq.(3) and ensemble averaging gives:

$$\partial_t \bar{f}_1 + \frac{\vec{p}}{E} \cdot \nabla_r \bar{f}_1 - \nabla_r \bar{U} \cdot \nabla_p \bar{f}_1 = \langle \nabla_r \delta U \nabla_p \delta f_1 \rangle \quad (5)$$

where one recognizes in the lhs the Vlasov term and in the rhs the Balescu-Lennard collision term [5,9,10]. The mean-field is given by:

$$\bar{U}(\mathbf{r}) = \frac{1}{N_{ev}} \Sigma_{ev} \Sigma_j \mathbf{V}(\mathbf{r}, \mathbf{r}_j) \quad (6)$$

In agreement to LQCD calculations[1,8] the interacting potential  $V(r)$  for quarks is ( $\hbar = 1$ ):

$$V(r_{ij}) = 3 \Sigma_{a=1}^8 \frac{\lambda_i^a \lambda_j^a}{2} \left[ \frac{8\pi}{33 - 2n_f} \Lambda (\Lambda r_{ij} - \frac{f(\Lambda r_{ij})}{\Lambda r_{ij}}) + \frac{8\pi}{9} \bar{\alpha} \frac{\langle \sigma_q \sigma_{\bar{q}} \rangle}{m_q m_{\bar{q}}} \delta(\mathbf{r}_{ij}) \right] \quad (7)$$

and[8]

$$f(t) = 1 - 4 \int \frac{dq}{q} \frac{e^{-qt}}{[\ln(q^2 - 1)]^2 + \pi^2} \quad (8)$$

We fix the number of flavors  $n_f = 2$  and the parameter  $\Lambda = 0.25 GeV$ . In Eq.(7) we have added to the Richardson's potential the chromomagnetic term (ct), very important to reproduce the masses of the hadrons in vacuum. Since in this work we will be dealing with finite nuclei, the ct can be neglected, we only notice that with the parameters choice discussed here, the hadron masses can be reproduced by suitably tuning the ct term [3].

The  $\lambda^a$  are the Gell-Mann matrices. From lattice calculations we expect that there is no color transport for distances of the order of  $0.2 - 0.3 fm$ , which are distances much shorter than the ones we will be dealing with in this paper. Thus we will use the  $\lambda_{3,8}$  only commuting diagonal Gell-Mann matrices (Abelian approximation)[7].

Numerically the BLV equation(5) is solved by writing the one body distribution function as:

$$\bar{f}_1(r, p, t) = \frac{1}{n_{tp}} \sum_i^N \sigma_i(t) \delta(r - r_i(t)) \delta(p - p_i(t)) \quad (9)$$

$N = Q n_{tp}$  is the number of such terms. Actually,  $N$  is much larger than the total quark number  $Q$ , so that we can say that each quark is represented by  $n_{tp} = N_{ev}$  terms called test particles(tp). Inserting Eq.(9) in the BLV equation gives the Hamilton equations of motion (eom) for the tp [4]. The weight factor fulfills the equation  $\partial_t \sigma_i(t) = \nabla_{r_i} \delta U \nabla_{k_i} \delta f_1$  [11]. Initially the weight factor is equal to 1 for the tp. When for a particular tp the weight factor becomes larger than 2 a new tp (quark) is created in the same phase space region. In the same way if the weight becomes less than zero for a given tp another tp with negative sign ( an anti-quark) is created. Notice that on average over time and ensembles the number of newly created quarks is equal to the number of anti quarks. Particular care is taken in order to have specific color conservation as well. We would like to stress now that in our picture the mean field and the collision term are calculated in a consistent

way starting from a two body potential. This is indeed a quite general method which can be applied to other physical systems.

Initially, we distribute randomly the tp in a sphere of radius  $R = r_{0B}A^{1/3}$  (the radius of the nucleus) in coordinate space and  $p_f$  in momentum space.  $r_{0B} = (\frac{3}{4\pi\rho_B})^{1/3}$ ,  $A = Q/3$  and  $\rho_B$  is the baryon density.  $p_f$  is the Fermi momentum estimated in a simple Fermi gas model by imposing that a cell in phase space of size  $h = 2\pi$  can accommodate at most two identical quarks of different spins, flavors and colors. A simple estimate gives the following relation between the quark density,  $n_q$ , and the Fermi momentum:

$$n_q = \frac{g_q}{6\pi^2} p_f^3 \quad (10)$$

The degeneracy number  $g_q = n_f \times n_c \times n_s$ , where  $n_c$  is the number of colors and  $n_s$  is the number of spins[2]. For quarks and anti quarks 3 different colors are used red, green and blue (r,g,b) [1].

### 3. Results

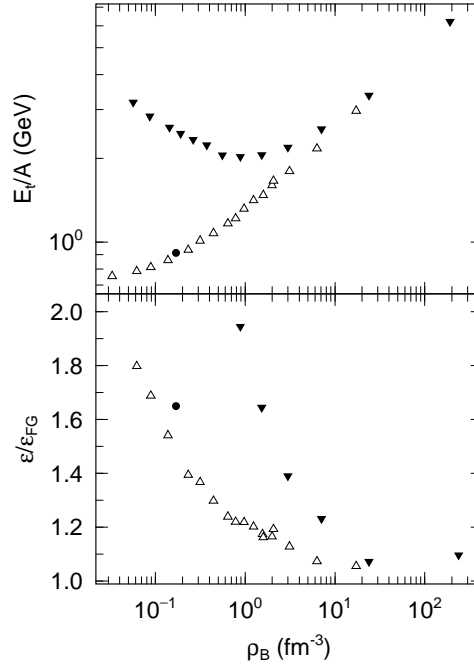


Figure 1. Energy per nucleon (top) and energy density of the quarks (bottom) vs. baryon density.

In figure(1), we plot the total energy per nucleon (top) and energy density (in units of the Fermi gas energy density[2]) vs. baryon density. The full triangles give the results obtained by randomly distributing the tp as described above. We notice that a minimum at about  $\rho_c = 2.08 fm^{-3}$  is found with  $E_t/A = 2GeV/A$ . Such a minimum is at much

higher density and energy than expected for the ground state (gs) of a nucleus ( $\rho_0 = 0.16 fm^{-3}$  and  $E_t/A = 0.938 - 0.016 GeV/A$ ). An important property of the system that we have described above is the following. If we rotate the quarks in color space, regardless of their position in r-space, the total energy will remain the same. This is indeed a "pure" Vlasov solution and demonstrates explicitly the in-capability of the Vlasov approach to give clustering of quarks into nucleons. However this result is already instructive since it gives us an hint on where the quark and nuclear matter are located, i.e. above and below  $\rho_c$  respectively. This result is qualitatively in agreement with the Hartree-Fock(HF) calculations of refs.[12,13] (compare to fig.1 in [12]). For a discussion on why the  $E_t/A$  increases at low densities in the HF/VE approaches we also refer to [12].

Of course distributing randomly the quarks in a sphere in r and p-space is not the most economical way to find the real gs of the system. In MD one searches for a minimum energy by introducing a friction term. The friction acts in such a way to lead the particles to a configuration for which the potential energy is a minimum. We cannot use this technique for our system since we are dealing with fermions and the friction term will destroy the initial antisymmetrization. In order to overcome this difficulty we adopt the following strategy. First we prepare  $N_{ev}$  events as above, and we evolve them numerically solving the eom but with friction included. Because of the large number of particles interacting with attractive and repulsive forces, the quarks will slowly evolve to new positions where the potential is lower while keeping the initial root mean square radius approximately constant. When the averaged potential (over events) reaches a given value  $V_{min}$ , we look for the two closest particles  $(j, k)$  to a quark  $i$  in the same event. For these three quarks we know what the local density and the number of colors are. For instance if in a certain region we find two red and one blue quark, we use  $n_c = 2$  in Eq.(10) and calculate the local density from the knowledge of the distances of the 3 quarks. In this way the Fermi momentum is defined locally similar to the procedure used in nuclear or atomic physics (Thomas-Fermi approximation)[14]. We repeat this for all quarks  $i$  in all events and calculate the total energy for this state. We let the system evolve again with friction included to a lower potential energy  $V_{min} = V_{min} - \delta V$ , where  $\delta V$  is a constant. We calculate the local density and local color numbers again and apply the Thomas-Fermi approximation to obtain the new total energy. We stop the procedure when the total energy is a minimum. The initial conditions so obtained are then propagated in time using the VE in order to maintain the initial antisymmetrization.

The open triangles in fig.(1) are the result of the minimization. Notice especially at low densities the large decrease of the total energy of the system as compared to the Vlasov result. Now the calculated total energy at the nuclear gs is very close to the experimental value indicated by the full circle. However, we find slightly lower energies at lower densities, i.e. the gs of the nucleus is shifted in our calculations at about 1/10 of the experimental one. This should not be surprising in view of the simple potential that we have used. Also we have not tried to look for a best set of fitting parameters in these exploratory studies. At low  $\rho_B$ , the global invariance for rotations in color space is lost, i.e. if we exchange the colors of two *distant* quarks, the total energy of the system will change. At high densities (larger than  $2 fm^{-3}$ ) the Vlasov and MD solutions are the same. This can be also seen in the bottom part of the figure where energy densities are given. We would like to stress again the qualitative agreement to ref.[12] where a

stochastic method had been used to calculate the g.s. energy of the system. This is quite evident if one compares our fig.1 to fig.1 of [12].

The energy density displayed in fig.1 (bottom) is a smooth function apart some fluctuations around  $2fm^{-3}$  density. From this result we can exclude a first order phase transition but a second order phase transition might be possible, see the discussion in [15].

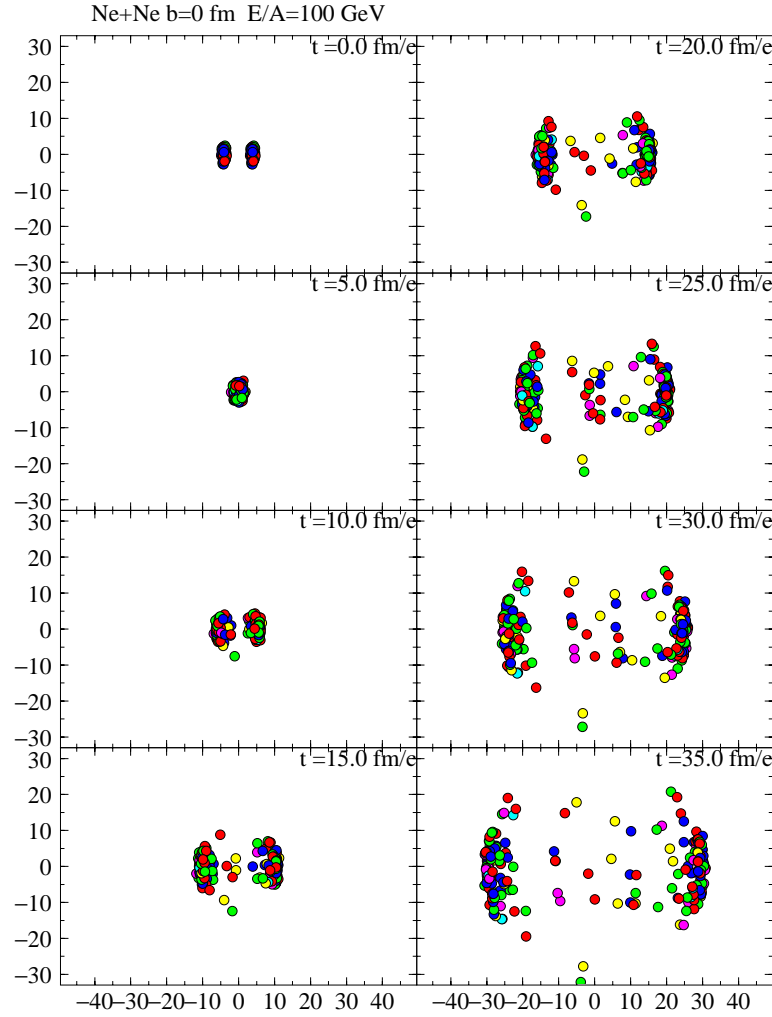


Figure 2. Ne+Ne collision at 100 GeV/A and  $b=0$ fm. The quarks are given by the (r,g,b) while the created anti quarks by the yellow, magenta and cyan colors.

Once we have devised a method to find a (approximate) nucleus ground state starting from quark degrees of freedom we can perform nucleus-nucleus collision simulations. In figure (2) we plot snapshots of a simulation for Ne+Ne collision at 100 GeV/A. Initially the

quarks are distributed randomly in a sphere of radius  $R$  and the minimization procedure is applied. Such an initial condition is Lorentz boosted and the dynamics is followed in time. In this simulation  $N_{ev} = 9$  for numerical reasons and one event only is displayed in the figure (2). Because of the large fluctuations from event to event, the collision term is very important and new quarks and anti quarks are created. These are displayed in the figure with different colors. The newly created quarks are strongly sensitive to the parameters entering the two body force and especially to the linear term. This might have some connection with the Schwinger particle production mechanism [2] but more studies are needed to find a precise link (if any).

#### 4. Summary

In conclusion in this work we have discussed microscopic Vlasov/MD approaches to finite nuclei starting from quark degrees of freedom with colors. In order to obtain the correct initial conditions we have introduced a method based on MD with frictional cooling plus a Thomas-Fermi approximation for the Fermi motion. We have shown that the method is able to describe at least qualitatively the well known features of nuclei near the ground state. At high densities a second order phase transition from nuclear to quark matter is predicted. Such a transition is due to the restoration of global color invariance at high densities and we have defined an order parameter accordingly[15]. We have used the initial conditions for the nucleus ground state to simulate heavy ion collisions at ultra-relativistic energies with the Balescu Lenard collision term. The mean field and the collision term are calculated starting from the same elementary interactions. Collisions are connected to deviations from the average mean field. This gives also large fluctuations in the distribution function which results in new particles creation. Our approach can be very useful for the understanding of the quark gluon plasma formation and its signatures.

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#### REFERENCES

1. B.Povh, K.Rith, C. Scholz, F.Zetsche, *Particles and Nuclei: an introduction to the physical concepts*, Springer, Berlin, 1995.
2. C.Y. Wong, *Introduction to High-Energy Heavy ion Collisions*, World Scientific Co., Singapore,1994; L.P. Csernai *Introduction to Relativistic Heavy ion Collisions*, John Wiley and Sons, New York, 1994.
3. A. Bonasera "Quark Dynamics on Phase Space", nucl-th/9905025, "Vlasov Dynamics of Dense Quark Matter", nucl-th/9908036, Phys.Rev.**C60**(1999)65212.
4. A. Bonasera, F. Gulminelli and J. Molitoris, Phys. Rep. **243**, (1994)1; G. Bertsch, S. Dasgupta, Phys. Rep.**160**(1988)189, and references therein.
5. E.M. Lifshitz and L.P. Pitaevskii, *Physical Kinetics*, Pergamon Press 1991.
6. T.Vetter, T.Biro and U.Mosel, Nucl.Phys.**A581**(1995)598; S.Loh, T.Biro, U.Mosel and M.Thoma, Phys.Lett.**B387**(1996)685; S.Loh, C.Greiner, U.Mosel and M.Thoma, Nucl.Phys.**A619**(1997)321; S.Loh, C.Greiner and U.Mosel, Phys.Lett.**B404**(1997)238; C.T. Traxler, U. Mosel and T.S. Biro Phys. Rev. **C59**(1999)1620.

7. M.Hofmann et al. e-print nucl-th/9908030.
8. J.L. Richardson, Phys.Lett. **82B**(1979)272.
9. M.Belkacem, V.Latora and A. Bonasera, Phys.Lett.**326B** (1994)21.
10. D.F.Litim and C.Manuel, Phys.Rev.Lett**82**(1999)4981.
11. A.Bonasera, V. Kondratiev, A. Smerzi and A.E. Remler Phys.Rev.Lett. **71**(1993)505.
12. C.J.Horowitz, E.J. Moniz and J.W.Negele, Phys. Rev. **D31**(1985)1689.
13. G.Roepke, D.Blaschke and H.Schulz, Phys. Rev. **D34**(1986)3499.
14. P.Ring, P.Schuck, *The Nuclear Many-Body Problem*,Springer-Verlag, New York, 1980.
15. A.Bonasera, e-print nucl-th/0003020